

Timelike form factors at high energy

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Abstract

The difference between the timelike and spacelike meson form factors is analysed in the framework of perturbative QCD with Sudakov effects included. It is found that integrable singularities appear but that the asymptotic behavior is the same in the timelike and spacelike regions. The approach to asymptotia is quite slow and a rather constant enhancement of the timelike value is expected at measurable large Q^2 . This is in agreement with the trend shown by experimental data.

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1 Introduction

There is now a long history of continuous progress in the understanding of electromagnetic form factors at large momentum transfer. After the pioneering works [1] leading to the celebrated quark counting rules, the understanding of hard scattering exclusive processes has been solidly founded by Brodsky and Lepage [2].

A perturbative QCD subprocess scaling like $\alpha_S(Q^2)/Q^2$ in the simplest case of the meson form factor is factorized from a wave function-like distribution amplitude

$$\varphi(x, Q^2) = \int^Q \psi(x, \mathbf{k}_T) d\mathbf{k}_T$$

(x being the light cone fraction of momentum carried by the valence quark), the Q^2 dependence of which is analysed in the renormalization group approach. Although an asymptotic expression emerges from this analysis for the x dependence of the distribution:

$$\varphi_{as} \propto x(1-x)$$

in the meson case, it was quickly understood that the evolution to the asymptotic Q^2 is very slow and that indeed some non perturbative input is required to get reliable estimates of this distribution amplitude at measurable Q^2 . Thanks to the QCD sum rule approach, such a function was proposed by Chernyak and Zhitnitsky [3], which were followed by other model dependent proposals [4, 5].

These developments helped theoretical estimates to get closer to real experimental data but a severe criticism [6] remarked that most of the contributions to the form factor were coming from end-point regions in the x integration, especially when very asymmetric distribution amplitudes such as those of [3] were used. This is not welcome since one may doubt the validity of the perturbative calculation in these regions. The recent work of Li and Sterman [7] solves this problem by proposing a modified factorization formula which takes into account Sudakov suppression of elastic scattering for soft gluon exchange. This inclusion leads to an enlargement of the domain of applicability of (improved) perturbative QCD calculations of exclusive processes.

The case of timelike form factors has not been much studied theoretically [8, 9]. Experimental data on the proton magnetic form factor $G_M(Q^2)$ [10] show a definite difference between the spacelike and timelike values at the highest measured Q^2 . A recent analysis of the $\psi \rightarrow \pi\pi$ decay [11] leads to a similar problem for the pion form factor at $O(10 GeV^2)$ transfer. Note, however, that the experimental extraction of the spacelike form factor has been recently suspected to suffer from large uncertainties [12].

In this paper, we carefully analyse in the Li-Sterman framework [7] the ratio between high Q^2 timelike and spacelike meson form factors. Not surprisingly, we find that this ratio goes asymptotically to 1 but we show that this approach to asymptotia is slow and that factors of the order of 2 follow at measurable Q^2 from reasonable assumptions on wave functions.

2 The spacelike form factor

In this section, we review the formalism as it has been developed for the spacelike case.

2.1 Hard scattering picture

The spacelike form factor measures the ability of a pion to absorb a virtual photon (carrying a momentum q with $q^2 = -Q^2 < 0$) while remaining intact. It is defined by the formula:

$$\langle \pi(p') | J^\mu | \pi(p) \rangle = e_\pi F(Q^2) (p + p')^\mu, \quad (1)$$

where e_π is the pion electric charge and momenta are defined in Fig. 1.

In the hard scattering regime, that is when Q is very high with respect to the low energy scales of the theory (the QCD scale Λ and the pion mass), Brodsky and Lepage have motivated the following three step picture for the process, valid in the light-front formalism [13, 2]:

- the pion exhibits a valence quark-antiquark “soft” (see below) state,
- which interacts with the hard photon leading to another soft state,
- which forms the final pion.

This leads to the convolution formula:

$$F(Q^2) = \psi_{in} * T_H * \psi_{out}^* \quad (2)$$

and the graphical representation of Fig. 2.

The most important feature of this picture is that it separates hard from soft dynamics. The amplitude T_H , the *interaction*, reflects the hard transformation due to the absorption of the photon and is hopefully calculable in perturbative QCD, because the effective couplings are small in this regime due to the asymptotic freedom. The amplitude ψ , the *wave function*, which depends on low energy dynamics is outside of the domain of applicability of perturbative QCD and is, at present, far from being fully understood from the theory. It is however process independent and contains much information on confinement dynamics. Factorization proofs legitimate this picture [14].

2.2 Infrared corrections

The need of a careful factorization is due to the infrared behavior of QCD: technically, large logarithms ($\sim \ln(Q/\lambda)$) appear in the renormalized one-loop corrections to naive “tree-graph” (λ is some infrared cut off needed to regularize soft and/or collinear divergences). As in the renormalization procedure, if factorization holds, these large corrections should be absorbed, here in the re-definition of the wave function. The proof of factorization and its consequences upon the wave functions are studied in the pattern of the renormalization group. Without entering into a detailed discussion, let us sketch

the procedure (see [15] for more on this leading logarithms calculation and also for the renormalization group treatment).

The first step is to compute the naive hard amplitude, that is consider the tree graph of Fig. 3, and the three other graphs related to it by C and T symmetries. One finds, with notations explained on Fig. 3:

$$T_H = 16\pi\alpha_S C_F \frac{xQ^2}{xQ^2 + \mathbf{k}^2 - i\varepsilon} \frac{1}{xyQ^2 + (\mathbf{k} - \mathbf{l})^2 - i\varepsilon}, \quad (3)$$

where all quark momentum components are kept. Note that we have done the usual projection onto the pion S wave state: $\psi_\pi(p) \propto \frac{1}{\sqrt{2}}\gamma^5 \not{p}$ and used the C symmetry of the wave function. $C_F = 4/3$ is the color factor, while α_S is the QCD effective coupling at the renormalization point μ .

To examine one loop corrections to T_H , the relevant graphs to consider in light-cone gauge are those of Fig. 4.

They directly lead to the wave function correction, in the “double logarithms” or Sudakov region (namely: $\lambda \ll |\mathbf{q}| \ll u\frac{Q}{\sqrt{2}} \ll x\frac{Q}{\sqrt{2}}$, u and \mathbf{q} being respectively the light-cone fraction and transverse gluon momentum relatively to the pion):

$$\begin{aligned} \psi^{(1)}(x, \mathbf{k}) = & \frac{C_F}{2\pi^2} \int_{\lambda}^{xQ/\sqrt{2}} \frac{d^2\mathbf{q}}{\mathbf{q}^2} \alpha_S(\mathbf{q}^2) \int_{|\mathbf{q}|\sqrt{2}/Q}^x \frac{du}{u} \{ \psi^{(0)}(x-u, \mathbf{k} + \mathbf{q}) - \psi^{(0)}(x, \mathbf{k}) \} \\ & + \frac{C_F}{2\pi^2} \int_{\lambda}^{\bar{x}Q/\sqrt{2}} \frac{d^2\mathbf{q}}{\mathbf{q}^2} \alpha_S(\mathbf{q}^2) \int_{|\mathbf{q}|\sqrt{2}/Q}^{\bar{x}} \frac{du}{u} \{ \psi^{(0)}(x+u, \mathbf{k} + \mathbf{q}) - \psi^{(0)}(x, \mathbf{k}) \}, \end{aligned} \quad (4)$$

where $\bar{x} = 1 - x$ and the first term in the difference comes from vertex-like corrections and the second one from self energy ones; in the infrared region some partial cancellations occur between these corrections, but the cancellation is not complete.

To pursue this analysis, it is convenient to define the Fourier transform in the transverse plane:

$$\hat{\psi}(x, \mathbf{b}) = \int d^2\mathbf{k} e^{i\mathbf{k}\mathbf{b}} \psi(x, \mathbf{k}), \quad (5)$$

and to separate transverse and longitudinal variations of the wave function. One finds, omitting for the moment the second term in Eq. (4):

$$\begin{aligned} \hat{\psi}^{(1)}(x, \mathbf{b}) = & \frac{C_F}{2\pi^2} \left(\int \frac{d^2\mathbf{q}}{\mathbf{q}^2} \alpha_S(\mathbf{q}^2) (e^{-i\mathbf{q}\mathbf{b}} - 1) \int \frac{du}{u} \right) \hat{\psi}^{(0)}(x, \mathbf{b}) \\ & + \frac{C_F}{2\pi^2} \int \frac{d^2\mathbf{q}}{\mathbf{q}^2} \alpha_S(\mathbf{q}^2) e^{-i\mathbf{q}\mathbf{b}} \int \frac{du}{u} \left(\hat{\psi}^{(0)}(x-u, \mathbf{b}) - \hat{\psi}^{(0)}(x, \mathbf{b}) \right). \end{aligned} \quad (6)$$

This equation contains the typical corrections one has to consider in a hard process when dealing with either a big ($\gg 1/Q$) or a small ($\lesssim 1/Q$) neutral object.

2.3 Transverse behavior at large distance

The transverse behavior at large distance is driven by the first term of the previous equation, thanks to the vanishing of the summation with the oscillating components. This occurs when $b = |\mathbf{b}|$ is greater than at least a few times the inverse of the upper bound of the corresponding integral: $xQ/\sqrt{2}$. As a consequence, in the remaining expression, the infrared cut-off λ can be replaced by the natural one $1/b$, above which the vertex and self energy corrections do not compensate one another. Thus we get:

$$\hat{\psi}^{(1)} = -s(x, Q, b) \hat{\psi}^{(0)}, \quad s = \frac{C_F}{2\beta} \ln \frac{xQ}{\sqrt{2}} \left(\ln \frac{\ln xQ/\sqrt{2}}{\ln 1/b} - 1 + \frac{\ln 1/b}{\ln xQ/\sqrt{2}} \right), \quad (7)$$

with $\beta = (11 - \frac{2n_f}{3})/4$, n_f being the number of quark flavors. Here and in the following, it is understood that the energies and inverse separations are in the natural Λ_{QCD} unit.

We have kept the single log term which occurs in the integration, because it is the one necessary to express the true *dominant* large b suppression, which one obtains in a more complete treatment (that is leading and next to leading one) [7].

After the resummation of the ladder structure to all order, the above Sudakov factor exponentiates. Taking into account the term obtained with the substitution $x \rightarrow \bar{x} \equiv 1-x$, we get:

$$\hat{\psi}(x, b, Q) = e^{-s(x, b, Q) - s(\bar{x}, b, Q)} \hat{\psi}^{(0)}(x, b, Q). \quad (8)$$

Thus we get a strong suppression of the effective wave function as $b \rightarrow 1/\Lambda$, whatever the fraction x is, provided that Q is reasonably large.

The remaining object $\hat{\psi}^{(0)}$ is a soft component to start with. It is soft in the sense that it does not include loop-corrections harder than $1/b$. One may modelize it by including some b behavior or simply relate it to the distribution amplitude [15] setting:

$$\hat{\psi}^{(0)}(x, b) \approx \int_0^{1/b} \psi(x, \mathbf{k}) d\mathbf{k} = \varphi(x; 1/b). \quad (9)$$

2.4 Transverse behavior at small distances

The first term in Eq. (6) is negligible when the oscillating term remains close to 1 in the range of integration. This happens for b a few times less than $\max^{-1}(xQ, \bar{x}Q)$. In this case, soft divergences cancel one another and one finds:

$$\hat{\psi}^{(1)}(x) = \xi \frac{C_F}{2} \int_0^1 dx' \left\{ \frac{\hat{\psi}^{(0)}(x') - \hat{\psi}^{(0)}(x)}{x - x'} \theta(x - x') + \frac{\hat{\psi}^{(0)}(x') - \hat{\psi}^{(0)}(x)}{x' - x} \theta(x' - x) \right\}, \quad (10)$$

with the notation:

$$\xi = \frac{1}{\pi^2} \int_\lambda^Q \frac{d^2 \mathbf{q}}{\mathbf{q}^2} \alpha_s(\mathbf{q}^2) \sim \frac{1}{\beta} \ln \left(\frac{\ln Q}{\ln \lambda} \right). \quad (11)$$

We displayed this equation in a slightly different form than in the large b case to explicitly show that Eq. (4), in the limit of small b , is related to the distribution evolution proposed in [13].

Let us shortly review how this comes. Brodsky and Lepage had proposed a simpler factorization formula for exclusive processes [2]. It is easily derived from the previous treatment if one assumes that neither the wave function nor the hard amplitude give important contribution to the form factor when the transverse momenta are big. Neglecting all transverse momenta in T_H leads therefore to consider the k_T -integrated quantity:

$$\varphi(x) = \int d^2\mathbf{k} \psi(x, \mathbf{k}) \quad (12)$$

This distribution amplitude related to the wave function at $b = 0$ has a dependence in Q associated with the remaining collinear divergences in Eq. (10). Indeed, the exponentiated form of this convolution equation, once it is written for the distribution φ and generalized to other regions than the Sudakov one, leads to the celebrated expansion of $\varphi(x, Q)/x\bar{x}$ in a linear combination of a running logarithm together with a Gegenbauer polynomial. However, whereas this slow evolution is predictable, the expansion at some finite Q is inaccessible from perturbative reasoning.

3 The timelike form factor

3.1 Quark and gluon poles

In the timelike region, the hard process ruling $\gamma^* \rightarrow \pi^+\pi^-$ is drawn in Fig. 5 and the hard amplitude is simple to deduce from the spacelike formula (3) changing $p \rightarrow -p''$ or $Q^2 \rightarrow -W^2$, $W^2 = q^2$. The new feature with respect to the spacelike form factor is that the contour of transverse momenta integration now goes near poles located at either: $\mathbf{k}^2 = xW^2 + i\varepsilon$ or: $(\mathbf{k} - \mathbf{l})^2 = xyW^2 + i\varepsilon$.

These poles are automatically ignored in the pattern of the Brodsky Lepage formalism as being too far from the contributing region of integration. However, whereas this argument is reasonable at asymptotic regime, we can expect some consequences of the presence of these singularities when the energy is not so high.

Technically, these poles are, except in the end point regions ($x, y \rightarrow 0$), far from the bounds of integration of the two independent variables $k = |\mathbf{k}|$ and $K = |\mathbf{k} - \mathbf{l}|$. Therefore, we may evaluate the integral by deforming the contour of integration in the complex plane of each of these variables.

Another question one may worry about, is the physical origin of these poles. A complete physical amplitude, for example the form factor $F(Q^2)$, considered for complex value of Q^2 , has poles and cut along the real negative axis reflecting the existence of intermediate physical (on mass shell) states. These intermediate states are hadronic ones and therefore correspond to the “asymptotical” objects of confined QCD. The poles we encounter in our present computation, internal gluon or quark lines going on mass shell, of course, do not correspond to observable states. However they only appear in a differential amplitude which itself is not observable. Provided, this differential amplitude is integrable,

the resulting form factor will only contain, as a remainder of this kind of singularities, a real and imaginary parts which one would also expect in a purely hadronic computation.

3.2 Hard scattering amplitude in b space

The Sudakov suppression is likely to be important at timelike transfer, so it is necessary to take the transverse Fourier transform of the hard amplitude. Furthermore, whenever this is possible, it is interesting to get an expression not only for negative t or positive s , but also for complex values of the generalized transfer. Let us define $z = \sqrt{-t}$, $\arg(z) \in [-\frac{\pi}{2}, 0]$, so that in the spacelike side of the complex plane, we get: $z = Q$ whereas in the timelike side: $z = -iW$.

The expression of the form factor, with the Fourier transform of Eq. (3) and the replacement of Q by z , is:

$$F = 16\pi C_F \int dx dy \int b_1 db_1 \hat{\psi}(x, b_1) b_2 db_2 \hat{\psi}(y, b_2) \alpha_S T(b_1, b_2, x, y, z),$$

$$T = K_0(\sqrt{xy}zb_2) xz^2 \left\{ \theta(b_1 - b_2) I_0(\sqrt{x}zb_2) K_0(\sqrt{x}zb_1) + (b_2 \leftrightarrow b_1) \right\}, \quad (13)$$

where angular integrations have been done thanks to the cylindrical symmetry of both hard amplitude and S wave wave function. b_2 and $|b_1 - b_2|$ are the transverse distances of, respectively, the gluon vertex and the internal quark vertex. The functions K_0 and I_0 are modified Bessel functions of order 0, the first appearing in the equation comes from the gluon propagator, while the remainder comes from the quark propagator.

3.3 Asymptotic behavior

Bessel functions have different asymptotic behaviors in various directions of the complex plane: for $|\zeta| \rightarrow \infty$, $\arg(\zeta) \in [-\frac{\pi}{2}, 0]$ we have [16],

$$K_0(\zeta) \approx \sqrt{\frac{\pi}{2\zeta}} e^{-\zeta}, \quad I_0(\zeta) \approx \sqrt{\frac{2}{i\pi\zeta}} \cosh(\zeta + i\frac{\pi}{4}), \quad (14)$$

we thus have to study how the asymptotic dependence of the form factor is affected by this direction. In particular in the timelike limit, the integrand is no more exponentially suppressed.

There is, *a priori*, no general constraint to ensure that the limit of some observable like a form factor should be the same in every directions in the complex plane. Even though $F(z)$ is analytic and:

$$F(z) \propto \frac{\alpha_S(z^2)}{z^2} (1 + \varepsilon(z)), \quad (15)$$

with the limit: $\varepsilon(z) \xrightarrow[|z| \rightarrow +\infty]{\arg z = 0} 0$, our ignorance of the true form of ε prevents us from concluding when another direction is considered. However, because one expects that the same kind of physics underlies exclusive processes, we expect that, at least in an asymptotic regime, we should find, for the leading behavior, the overlapping of exactly the same soft and hard amplitudes.

As a first step in the understanding of the features of the whole form factor, we may evaluate the b integrals in an analytical way, putting a simple form for the wave function:

$$\hat{\psi}(b) = \theta(B - b), \quad (16)$$

which automatically provides a cut-off to avoid indefinite integral. We also forget here the possible running of the coupling with transverse distances (see subsection 3.4) which appears with Sudakov suppression, here ignored.

With these simplifications, one finds for the integral over b_1 and b_2 :

$$I = \int b_1 db_1 \hat{\psi}(b_1) b_2 db_2 \hat{\psi}(b_2) T(b_1, b_2, x, y, z)$$

$$I = \frac{1}{xy z^2} + \frac{\sqrt{B}}{z^{3/2}} f(x, y, zB), \quad (17)$$

with f a function that we refrain from quoting here due to its lack of interest, except for its generic behavior for large $xy|z|B$: $f \sim e^{-zB}$.

As long as we avoid the timelike limit, we find the following leading behavior for the integration in the transverse plane:

$$I \sim \frac{1}{xy z^2}, \quad (18)$$

which displays the expected selection of small configuration by the hard process.

In the timelike region, this is no more true, as we get a modified power dependence:

$$I \sim \frac{1}{W^{3/2}} \sqrt{B} e^{iWB}, \quad (19)$$

with the appearance of the size B , together with an oscillating factor. Of course, we suspect here that we have found essentially the limit of our model object; nevertheless, we may anticipate that some reminiscence of this rather different behavior will occur in the non asymptotic regime.

A source of modification to the above result is the transverse behavior of the wave function. The rectangular form which we have chosen above and its steep variation reduces the occurrence of cancellations expected with an oscillating integrand. If we were speaking of Fourier-transform we would say that a rectangular function has relatively large components at large momenta in comparison with any similar but smoother function.

The Sudakov behavior reviewed in the second section plays this role. However, due to the transfer dependence of the Sudakov factor, we must firstly face the problem of its analytic continuation [17].

Before turning to this analytic continuation, we rewrite the expression of s from Eq. (7) for spacelike transfer, in the form:

$$s(x, Q, b) = \frac{C_F}{2\beta} \ln \frac{xQ}{\sqrt{2}} (U - 1 - \ln U), \quad U = \frac{-\ln b}{\ln \frac{xQ}{\sqrt{2}}}, \quad (20)$$

to explicitly see that s increases rapidly with $U < 1$ at large Q (remember that if x is

small, we can always consider the \bar{x} -term in turn) so that the region of not too large suppression is $U \lesssim 1$, where we have:

$$s \approx \frac{C_F}{4\beta} \ln \frac{xQ}{\sqrt{2}} (U - 1)^2. \quad (21)$$

For large timelike transfer, setting $Q = -iW$ in Eq. (20), we get:

$$s(x, W, b) - s_{SL}(x, W, b) \approx i \frac{C_F \pi}{4\beta} \ln U - \frac{C_F \pi^2}{16\beta \ln \frac{xW}{\sqrt{2}}} \quad (22)$$

with s_{SL} and U the spacelike expression of Eq. (20). In the above equation, the real part is effectively small, while the imaginary part remains close to 0 in the region of intermediate suppression. We can therefore assume, in the asymptotic regime, the same scale dependence for the Sudakov factor and use the expression s_{SL} for our study.

To simplify our purpose, we will concentrate on the simpler case one gets by considering only the transverse behavior of the gluon propagator, that is setting the transverse momentum to $\mathbf{k} = \mathbf{0}$ in Eq. (3). As we will show, this does not alter the naive behavior we previously get. However when taking the Fourier transform, only one transverse distance remains, $\mathbf{b} = \mathbf{b}_1 = \mathbf{b}_2$ and after angular integration we are led to replace the integral I in Eq. (17) by the quantity I' :

$$I' = \int_0^{\Lambda^{-1}} b db K_0\left(\frac{1}{2}zb\right) e^{-4s(|z|, b)} \quad (23)$$

where we have limited our study to the $x = y = \frac{1}{2}$ case. Thanks to the Sudakov suppression, the upper bound of the integral is naturally $b = \Lambda^{-1}$.

For the Sudakov exponent, we consider the approximate expression of section 2 with the prescription of Li and Sterman [7] which is to set the exponential to unity in the region that should not be controlled by Sudakov evolution, here for $b < 1/|z|$. With these simplifications, we get:

$$I' = \frac{4}{W^2} \left(-1 - iK_1(-i) + \int_1^{W/2} u du K_0(-iu) e^{-4s(W, \frac{2u}{W})} \right) \quad (24)$$

which is to be compared with the expression without Sudakov correction:

$$I = \frac{4}{W^2} (-1 - i \frac{W}{2} K_1(-iW/2)) \quad (25)$$

We present in Fig. 6 the result of a numerical computation for both the real part (a) and the modulus (b) of the quantity

$$\Delta = \frac{W^2}{4} I + 1 \quad (26)$$

which dictates the deviation from the counting rule canonical result and compare it to the original quantity $\frac{W^2}{4} I + 1$. We observe that after an intermediate regime ($W < 20\Lambda$), the expressions including Sudakov suppression slowly decrease to 0 contrary to the case of the rectangular wave function.

Another feature we have omitted until now is the fact that the expression for the form factor is a superposition of amplitude with various fractions x, y weighted by smooth distributions. This should also modify the $W^{-3/2}$ -power law and we examine this possibility in Appendix A.

We conclude that the $W^{-3/2}$ behavior does not resist to the inclusion of any realistic b or x - y integration procedure. Indeed the dimensional counting rules are valid at timelike transfers and furthermore the form factors are asymptotically the same.

3.4 Comparison at intermediate energy

Let us now turn to the discussion of the ratio of timelike over spacelike form factors in the intermediate range. The complete integration formula is:

$$F = \int dx dy \int b_1 db_1 b_2 db_2 \hat{\psi}^{(0)}(x, b_1) \hat{\psi}^{(0)}(y, b_2) T_H e^{-S} \quad (27)$$

with the hard scattering amplitude from Eq. (13): $T_H = 16\pi\alpha_S T$. The integration range for the longitudinal fraction of momentum goes from 0 to 1, whereas transverse distances it go from 0 to $1/\Lambda$ thanks to the Sudakov suppression at large b .

For the numerical study, we take into account the one loop running of the QCD coupling in the hard scattering:

$$\alpha_S(t) = \frac{\pi}{\beta \ln(t^2/\Lambda^2)}, \quad t = \max(\sqrt{xy}|z|, b_1^{-1}, b_2^{-1}) \quad (28)$$

with the prescription for the renormalization point described in [18] and $\Lambda = 200 \text{ MeV}$. The Sudakov factor e^{-S} contains the corrections for the two wave functions together with the anomalous running of the four quark operator T_H [18].

The Sudakov factor should be analytically continued as discussed in sub-section 3.3 [17]. It turns out that this procedure leads to quite model-dependent results at non asymptotic transfers. This has to do with the truncation of the exponentiated expression and with the need to suspect the validity of the approach when the (real part of the) Sudakov exponent becomes positive, i.e. when Sudakov suppression turns to an enhancement. We leave to Appendix B a somewhat detailed discussion of these effects, the conclusion being that an extra modification of the timelike value may come from this continued Sudakov exponential, but that it is quite difficult to reliably quantify this statement. In the following, we will thus keep the Sudakov factor at its spacelike (real) value.

We used various forms for the soft wave function $\hat{\psi}^{(0)}(x, b)$ to test the sensitivity of the result to this input. One may consider wave functions without intrinsic transverse behavior ($\hat{\psi}^{(0)}(x, b) = \varphi(x)$) and use either the asymptotic form:

$$\varphi_{as}(x) = \sqrt{\frac{3}{2}} f_\pi x(1-x), \quad (29)$$

with $f_\pi = 133 \text{ MeV}$ the pion decay constant, the CZ form:

$$\varphi_{CZ}(x) = 5(1 - 2x)^2 \varphi_{as}(x), \quad (30)$$

or other expansions in terms of Gegenbauer polynomials like those proposed in [4, 5]. In the following, we will show some results for one form [5]:

$$\varphi_{FHZ}(x) = (.6016 - 4.659(1 - 2x)^2 + 15.52(1 - 2x)^4) \varphi_{as}(x) \quad (31)$$

As mentioned in section 2, the two last distributions have slow logarithmic evolution with Q . We ignore this evolution because it is quite insignificant in the range of energy we consider here.

Following [8, 19], one may also include some intrinsic transverse behavior. We tried the different forms of wave functions described in [19] and found only small differences for the behavior of the ratio. Therefore, we only quote here the sample form for which we will show some results in the following:

$$\hat{\psi}^{(0)}(x, b) = \varphi(x) e^{-b^2/4b_0^2} \quad (32)$$

which is a simple way to modelize the transverse behavior without a long set of parameters. $2b_0$ related to the valence state radius is proposed in [19]: $b_0^2 = 4.082 \text{ GeV}^{-2}$.

Figs 7–8 show our numerical results for the meson form factors in the large but non asymptotic $Q^2 = |q^2|$ region¹. In Fig. 7, $Q^2 |F_\pi(Q^2)|$ is plotted against Q^2 for both timelike and spacelike regions up to $Q = 50\Lambda$. The distribution considered is the asymptotic one, Eq. (29). The slow convergence of the timelike and spacelike quantities is manifest while the counting rule ($F_\pi \propto 1/Q^2$) is reasonably well describing the Q^2 dependence down to a few GeV^2 in both cases. The inclusion of the intrinsic b -dependence given by Eq. (32) (dashed lines) does not significantly modify the results.

In Fig. 8(a), the modulus of the timelike form factor (multiplied by Q^2) is shown for the three choices of distribution amplitudes: CZ form (solid line), asymptotic one (dashed line) and FHZ one (long-dashed line). The experimental data shown comes from Ψ decay [11]. Sudakov suppression has been included but no intrinsic b -dependence. Fig. 8(b) shows the ratio of the timelike to the spacelike form factors. This ratio is rather wave function independent and decreases very slowly to 1 from a value of around 2 in most of the experimentally accessible range.

Although this ratio turns out to be quite difficult to reliably extract from experimental data in the meson case, it is quite straightforwardly measured in the proton case. We will analyse the proton case in a forthcoming work. If we restrict to a simple quark-diquark picture, we would get a timelike to spacelike ratio quite similar to the one obtained here for the meson case, and thus understand the experimental value.

¹In this sub-section, as there is no confusion, we do not distinguish the absolute value of spacelike and timelike transfers.

4 Conclusion

In this paper, we showed that the difference between spacelike and timelike form factors at large accessible transfer is predictable from an improved perturbative QCD analysis. We understand at least qualitatively the enhancement of the timelike values at large but non asymptotic transfers as mostly due to the integrable singularities of gluon and quark propagators. This strengthens the faith in the applicability of perturbative reasoning at intermediate energies (above a few GeV) at least for semi-quantitative understanding of the strong interaction physics.

For the pion case, the uncertainties in the extraction of the spacelike form factor [12] show the need for another way to access this observable, the simplest one in exclusive scattering. We demonstrated that the usual formalism of Brodsky and Lepage has to be improved to account for the differences between timelike and spacelike regions in the energy range experimentally reachable. More experimental data are still needed to test our knowledge of the pion wave function.

The proton case is more interesting since the extraction of the spacelike form factor is without ambiguity. The comparison of spacelike and timelike form factors thus appears to be a good way to understand the hadronic wave function.

Many other hard exclusive processes dwell on timelike transfers. The $\gamma\gamma \rightarrow \pi\pi$, $p\bar{p}$ reactions at fixed angle for instance demand a more careful analysis than available now, not to speak of the difficult instances where pinch singular diagrams mix up, as in the ratio of $p\bar{p}$ to pp elastic scattering. More work needs to be done and experimentally tested before we know for sure that exclusive timelike reactions help us to understand confinement dynamics through the unraveling of hadron wave functions in their lowest Fock state.

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Appendix A

We give in this Appendix some arguments for the power suppression of the oscillating factor we get in sub-section 3.3. Again we will concentrate on the simple case one has when neglecting the transverse momentum in the quark propagator (see sub-section 3.3) and consider the integral analogous to I in Eq. (17):

$$I'' = \int_0^B b db K_0(\sqrt{xyz}b) = \frac{1}{xyz^2} - \frac{B}{\sqrt{xyz}} K_1(\sqrt{xyz}B). \quad (33)$$

As in section 3, we should worry about the behavior of the Bessel function with large argument (K_1 has the same asymptotic behavior as K_0). We get:

$$I'' \approx \frac{1}{xyz^2} - \frac{\sqrt{\pi B/2}}{(\sqrt{xyz})^{3/2}} e^{-\sqrt{xyz}B}. \quad (34)$$

In this explicit asymptotic form, one may guess that because the phase varies rapidly with x or y due to the presence of the large $|z|$ factor there may be some destructive interferences when integrating the second term of I'' . The presence of any smooth distribution as weight functions will not destroy this feature. To see this explicitly we can perform the integration over some finite range for x to avoid region where the asymptotic form fails and also to allow further simplifications. Precisely, we look at:

$$J = \int_{1/2-a}^{1/2+a} dx \varphi(x) I''(x, y = \frac{1}{2}) \quad (35)$$

and replaces $x = 1/2$ everywhere in the integrand except in the phase where we take the expansion of the square root of x around $1/2$ up to first order. With these simplifications, we easily get:

$$J = \frac{8a}{z^2} \varphi(\frac{1}{2}) \left[1 - \sqrt{\frac{\pi}{zB}} e^{-\frac{zB}{2}} \frac{\sinh \frac{zBa}{2}}{a} \right], \quad (36)$$

which with the replacement $z = -iW$ has a modified behavior compared to I'' and the leading behavior is now identical in the spacelike and timelike directions of the complex plane. However, the previous equation, even if approximatively, still indicates qualitatively that this identical asymptotic behavior may be reached rather slowly.

Appendix B

We discuss in this Appendix the analytic continuation of the Sudakov suppression factor.

In the complete expression for the form factor Eq. (27), two factors: T_H and e^{-S} , are scale dependent and must *a priori* be analytically continued. In sub-section 3.3, we give some arguments to show that the Sudakov factor has a leading behavior which is not affected by analytic continuation so that we can study the contribution to timelike form factor ignoring this kind of difficulties. However, in the range of transfers we consider in the numerical study of sub-section 3.4, these arguments may not apply. In e^{-S} , the scale always appears in logarithms and after analytic continuation, one gets a phase which is sub-leading compared to the remaining large logarithm. The Sudakov exponent is known [15] up to next-to-leading logarithms and we may keep the imaginary part which comes from the leading-log (Eq. (22)) as a next-to-leading component. In this kind of analysis, the additional real part due to the product of logarithms (the π^2 -factor), which may lead to an additional enhancement [17], is automatically dropped. A further analysis of the amount of correction which may be provided by such terms gives an $O(10\%)$ extra enhancement of the timelike form factor.

The effect of the imaginary part in the Sudakov exponent appears to be more important. The ratio of the timelike to spacelike factor is:

$$\frac{e^{-S_{TL}}}{e^{-S_{SL}}} = e^{i(\phi(x)+\phi(1-x)+(x \leftrightarrow y))},$$

$$\phi(x, W, b) \approx -\frac{C_F \pi}{4\beta} \ln \frac{-\ln b}{\ln \frac{xW}{\sqrt{2}}}. \quad (37)$$

We must define a prescription to cut-off the small b region. For the spacelike Sudakov suppression, Li and Sterman [7] choose to include the exponential factor only in the region of large b defined by $bxW > \sqrt{2}$ and furthermore only when the sum of all exponents is negative and leads effectively to a suppression. The first prescription is associated with approximations discussed in section 2 and we will not question it in the following. In the spacelike case, it appears that the second constraint may be easily relaxed in a numerical study, because only a very small enhancement results when forgetting this constraint. We have observed that this is not likely to be the case for the timelike form factor.

A numerical study, in the range of transfer $10-30\Lambda$, with the consideration of the total gluon propagator alone and the prescription: ϕ from Eq. (37) if $\text{Re}(-S) < 0$, shows, for the asymptotic distribution, a 5–10% depletion of the timelike form factor with respect to the value without consideration of phase, whereas for the CZ distribution, the diminution is 20–30%.

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Figure 1: The pion form factor.

Figure 2: Factorization of the form factor.

Figure 3: Tree graph for T_H .

Figure 4: Leading radiative corrections grouped in the wave function.

Figure 5: The timelike hard amplitude.

Figure 6: The real part (a) and modulus (b) of the deviation to spacelike scaling Δ . Solid (dashed) line is with (without) the Sudakov suppression factor. W is in Λ units.

Figure 7: Timelike and spacelike form factors: different transverse behaviors. The x -dependence is the asymptotic form of Eq. (29). Energies are in Λ units.

Figure 8: Timelike form factor (a) and timelike over spacelike ratio (b): sensitivity to distribution. CZ is Eq. (30), FHZ (31), as (29); experimental data from [11].

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